Inequalities-II.

Problem 1. Deduce the general AM-GM inequality from the rearrangement inequality.

Problem 2. Let (x, y) be a point on the unit circle $x^2 + y^2 = 1$. Find the maximal and the minimal possible value of $3x + 4y$.

Problem 3. Prove that for any positive numbers a, b, c

$$\frac{ab}{a+b} + \frac{bc}{b+c} + \frac{ac}{a+c} \leqslant \frac{3(ab+bc+ac)}{a+b+c}.$$

Problem 4. Let $a_i > 0, s = a_1 + \cdots + a_n$. Prove, that

$$\frac{a_1}{s-a_1} + \cdots + \frac{a_n}{s-a_n} \geqslant \frac{n}{n-1}.$$

Problem 5. Let $c_1, \ldots, c_n$ be distinct natural numbers. Prove, that

$$c_1 + \frac{c_2}{2} + \cdots + \frac{c_n}{n} \geqslant n.$$

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